# Displacement Calculation Method of Super-long Telescopic Jib Structure Considering the Effect of Self Weight and Large Deformation

Gening Xu*[1], Guangheng Gao[2]

*[1, 2]College of Machinery Engineering, Taiyuan University of Science and Technology, Taiyuan 030024, China

*[1]xugening@sina.com; [2]ggh678@126.com

***Abstract***

Aiming at reducing weight of equipment, saving material and increasing lifting capability of crane, variable stiffness structure is designed in project. Ccumbersome calculation has increased the design difficulty of variable stiffness structural. For large slenderness ratio telescopic jib structure, the existing calculation methods simplify self weight as concentrated load and assume obeying small deformation criterion. For flexural deformation, the effect of self weight different distribution and large deformation caused by nonlinearity can not be ignored. Based on the effect of self weight different distribution and large deformation, a mathematical model of telescopic jib displacement was established and deduced out a formula of endpoint displacement of large slenderness ratio telescopic jib structure. In addition, large-deformation finite element method is used as a criterion to evaluate the correctness and precision of theoretical calculation formula.

***Keywords***

*Variable Stiffness; Large Slenderness Ratio; Telescopic Jib; Gravity Effect; FEM; Large-deformation*

## Introduction

Telescopic jib of truck crane is an important load-bearing component, whose function directly affects the working performance of the whole machine, and material is applied to improve the working performance of telescopic jib as far as possible, based on equivalent-strength design. In general, structure stiffness changes with the changing structural internal force, so variable stiffness structure is designed. The difficulty of variable stiffness structural design is increased by complicated and cumbersome calculation. At present, there are two groups of typical method to calculate the displacement of variable stiffness component: analytic method and finite element method. Analytic method needs to build simplified calculation model, and uses mathematical method to get the result, and the degree of difficulty to get the result can vary with the degree of simplified model, different types of constraints, the verbosity of primitive function, and so on. With the help of finite element theory and mature finite element analysis software, analysis accuracy is improved by establishing model that is more accurate and closer to the reality. Finite element method has been widely used in the mechanical engineering field. Reference to the present document, there isn't study of researching the different working way of self weight. For large slenderness ratio telescopic jib, different working ways of self weight lead to different flexural deformation and different stiffness of different jibs increases the effect of self weight on jib deformation. Therefore, the different working ways of self weight can not be ignored in the design process of variable stiffness structure. Displacement calculated method is studied considering the effect of self weight working ways and nonlinear deformation. A calculating formula of endpoint displacement of telescopic jib structures is deduced, in addition, large deformation finite element calculation method is used as criterion to evaluate the correctness and precision of formula in this paper.

## Calculation Considering Weight Effect

Variable cross-section jib structure is a biaxial bending component. The bending stress is more significant, so it is usually treated as cantilever jib. The simplified analysis model of telescopic jib structure considering weight effect is demonstrated in Fig 1. In the luffing plane, the deflection curve slope of the point between jib luffing hinge point and upper hinge point of hydraulic cylinder is near zero, so converted basic jib length can be expressed as

$$L_1 = L - l_1 / 2 \quad , \tag{1}$$

Where $L_1$ is the length of basic jib and $l_1$ is the length between jibs luffing hinge point and upper hinge point of hydraulic cylinder.

The inertia moment of every jib section $I_i$, the length of each jib $L_t$ and the serial number $i$ are indicated in Fig 1.

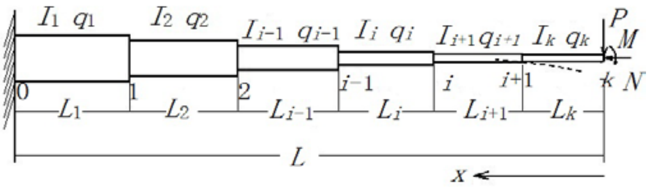


FIG 1 STRUCTURE MODEL CONSIDERING GRAVITY EFFECT

There are concentrated force $P$, axial force $N$ and bending moment $M$ effect on the $k$ point shown in Fig 1. There is an implicit assumption that the load $q_i$ of each jib is evenly distributed along the whole length of each jib respectively. Almost all the slenderness ratio of telescopic jib are larger than 30, so telescopic jib belongs to compression-bending column. Deformation of jib caused by bending moment and axial force can not be neglected. According to literature 8, the deformations respectively caused by concentrated force, axial force and bending moment can be solved and the total deformation of endpoint of the jib can be gained, based on superposition principle.

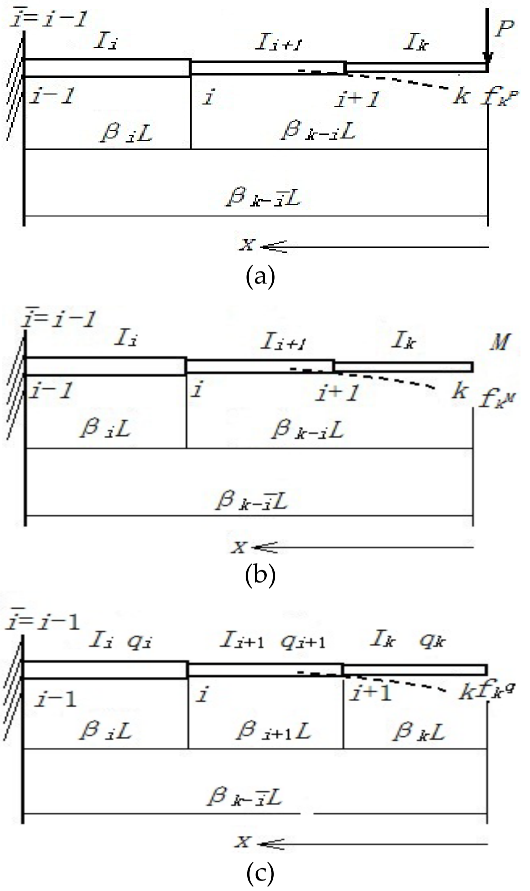


FIG 2 MODEL OF CALCULATION

To calculate the total deformation, three models are abstracted from Fig 1, as expressed in Fig 2, considering the effect of $P$, $M$ and $q_i$ separately.

The $i-1$ point is processed into fixed end and substitute $\bar{i}$ for $i-1$.

### *Deformation Calculation Under External Load*

According to literature 2, the deformation of top jib respectively caused by concentrated force and bending moment can be described as

$$f_k^P = \frac{PL^3}{3E}\sum_{i=1}^{k}\frac{1}{I_i}(\beta_{k-\bar{i}}^3 - \beta_{k-i}^3) \tag{2}$$

$$f_k^M = \frac{ML^2}{2E}\sum_{i=1}^{k}\frac{1}{I_i}(\beta_{k-\bar{i}}^2 - \beta_{k-i}^2) \tag{3}$$

### *Deformation Calculation Under Weight Load*

Apply a per unit concentrated force to the deformation direction, based on principle of virtual work, in the course of the calculation of the geometric linear displacement of structure. Internal-force equation of each segment in virtual state can be expressed as

$$\begin{aligned} segment, i &: \overline{M} = -x \\ segment, i+1 &: \overline{M} = -x \\ segment, k &: \overline{M} = -x \end{aligned} \tag{4}$$

Internal-force equation of each segment in actual state can be expressed as

$$\begin{aligned} segment.i: \quad M_P &= -[0.5q_i(x-\beta_{i+1}L-\beta_k L)^2 \\ &+ q_{i+1}\beta_{i+1}L(x-0.5\beta_{i+1}L-\beta_k L) \\ &+ q_k\beta_k L(x-0.5\beta_k L) \\ segment.i+1: M_P &= -[0.5q_{i+1}(x-\beta_k L)^2 \\ &+ q_k\beta_k L(x-0.5\beta_k L) \\ segment.k: \quad M_P &= -0.5q_k x^2 \end{aligned} \tag{5}$$

Treat the structure in Fig 2.(c) as a uniform cross-section cantilever beam with equal section inertia $I_i$. Deformation caused by the acting force of segment $i$ can be derived from

$$\begin{aligned} f_i^q &= \sum\int\frac{\overline{M}M_P ds}{EI} \\ &= \int_{(\beta_{i+1}+\beta_k)L}^{\beta_{k-\bar{i}}L}\frac{1}{EI_i}\{0.5q_i[x-(\beta_{i+1}+\beta_k)L]^2 \\ &+ q_{i+1}\beta_{i+1}L[x-(0.5\beta_{i+1}+\beta_k)L] \\ &+ q_k\beta_k L(x-0.5\beta_k L)\}xds \\ &+ \int_{\beta_k L}^{(\beta_{i+1}+\beta_k)L}\frac{1}{EI_{i+1}}[0.5q_{i+1}(x-\beta_k L)^2 \\ &+ q_k\beta_k L(x-0.5\beta_k L)]xds \\ &+ 0.5\int_0^{\beta_k L}\frac{q_k x^3}{EI_k}ds \end{aligned} \tag{6}$$

Self weight $q_i$ is the mass sum of the $ith$ jib mainly in three parts: the jib itself, telescopic mechanism weight and the attached parts weight. And the load is assumed to be even-distribution along the whole jib. This relational expression can be expressed as

$$q_i = \frac{m_i g}{\beta_i L} = \frac{G_i}{\beta_i L} \tag{7}$$

Substituting Eq.(7) into Eq.(6) gives

$$\begin{aligned} f_i^q &= \sum \int \frac{\overline{M} M_P ds}{EI} \\ &= \int_{(\beta_{i+1}+\beta_k)L}^{\beta_{k-i}L} \frac{1}{EI_i} \{ \frac{G_i[x-(\beta_{i+1}+\beta_k)L]^2}{2\beta_i L} \\ &+ G_{i+1}[x-(0.5\beta_{i+1}+\beta_k)L] + G_k(x-0.5\beta_k L)\} x ds \\ &+ \int_{\beta_k L}^{(\beta_{i+1}+\beta_k)L} \frac{1}{EI_{i+1}} [\frac{G_{i+1}(x-\beta_k L)^2}{2\beta_{i+1}L} + G_k(x-0.5\beta_k L)]ds \\ &+ q_k \beta_k L(x-0.5\beta_k L)] x ds \\ &+ \int_0^{\beta_k L} \frac{1}{EI_k} \frac{G_k x^3}{2\beta_k L} ds \end{aligned} \tag{8}$$

According to table.11-2 of literature .8, the following equation is always right as well the jib section number is not larger than five, as shown in Eq.(9)

$$\beta_1 \approx \beta_2 = \beta_i = \beta_{i+1} = \beta_k \tag{9}$$

Substituting $l_1 = 0.5L_1$ into Eq.(1) and substituting Eq.(9) into Eq.(8)

$$\begin{aligned} f_i^q &= \sum \int \frac{\overline{M} M_P ds}{EI} \\ &= \int_{2\beta_k L}^{\beta_{k-i}L} \frac{1}{EI_i} [\frac{G_i \beta_k L}{2} (\frac{x}{\beta_k L} - 2)^2 \\ &+ G_{i+1}\beta_k L(\frac{x}{\beta_k L} - \frac{3}{2}) + G_k \beta_k L(\frac{x}{\beta_k L} - \frac{1}{2})] x ds \\ &+ \int_{\beta_k L}^{2\beta_k L} \frac{1}{EI_{i+1}} [\frac{G_{i+1}\beta_k L}{2} (\frac{x}{\beta_k L} - 1)^2 \\ &+ G_k \beta_k L(\frac{x}{\beta_k L} - \frac{1}{2})] x ds \\ &+ \int_0^{\beta_k L} \frac{1}{EI_k} \frac{G_k x^3}{2\beta_k L} ds \end{aligned} \tag{10}$$

The equation shows that the highest time of integrand is no larger than three, thus its primitive function is easy to get as well as the value of displacement $f_i^q$. The algebraic sum of Eq.(2), Eq.(3) and Eq.(10) is the total displacement of jib.

## Engineering Example

Take a three-section telescopic box jib for example. The cable length is 30 m, and the upper hinge point of hydraulic cylinder is set to be the middle position of the basic jib. According to the supporting mode of jib, its calculated model can be simplified as shown in Fig 3. Geometric property data of the jib is listed in Tab 1.

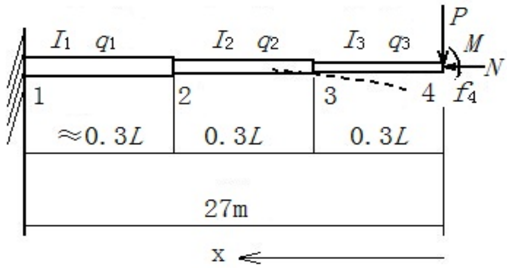


FIG.3 MODEL OF CALCULATION

TABEL.1 GEOMETRIC PROPERTY DATA OF BOOM

| jib | Length L/mm | Sectional Area /mm2 | Inertia Iy/mm⁴ | Slender Ratioλ | Mass m/kg | Calculation Force G/N |
|---|---|---|---|---|---|---|
| 1 | 12000 | 1.77E+04 | 1.79E+09 | 56.16 | 2092.14 | 30754.46 |
| 2 | 11250 | 1.63E+04 | 1.40E+09 | 57.75 | 1806.24 | 26551.78 |
| 3 | 11250 | 1.48E+04 | 1.07E+09 | 63.87 | 1640.03 | 24108.37 |

The model is simplified by converting other force except self weight into concentrated force and applied on the jib tip point. In the luffing plane, the axial force is 60000 N; the horizontal force is 10000 N; and the bending moment is 40000 N-m. The force of the slewing plane is excluded from discussion in this paper.

### *Treat Self Weight As Concentrated Weight*

Some approximation is made in literature 8 that one-third of the self weight has effect on the jib tip point, and the rest has effect on the jib root.

1) ***Equivalent Inertia Moment Method***

The maximum displacement of jib in the luffing plane is 576.59 mm, by using equivalent inertia moment method.

2) ***Small Deformation FEA of Concentrated Weight***

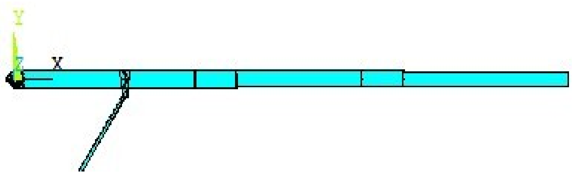


FIG 4 FEM MODEL OF TELESCOPIC BOOM

Small deformation finite-element analysis is a finite element method without considering the impact of the additional blending moment. By using

Beam188 element, a finite element model of this jib structure is constructed and analyzed in finite element software shown in Fig 4.

The jib foot is hinged at the slewing platform by hinged pin shaft and the jib median is hinged with upper point of the luffing-hydraulic cylinder. So both three degree-of-freedom in three coordinates and rotation about $x$ and $z$ coordinate axes should be constrained, as well as the $y$-translational DOF of the upper point. One-third of its weight is applied on the jib tip point while the rest on the jib root. The additional bending moment caused by self weight is not considered. The analysis results are shown in Fig 5.

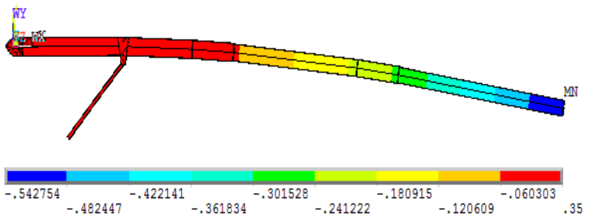


FIG 5 RESULT OF SMALL DEFORMATIN FEA

The result shows that the maximum displacement of jib in the luffing plane is 542.89 mm.

### 3) *Large Deformation FEA of Concentrated Weight*

Unlike small deformation finite-element analysis, the impact of the additional blending moment is considered in large deformation finite element analysis. External loads are applied step by step, and the sub-step number can be set based on the accuracy of result. The analysis results are shown in Fig 6.

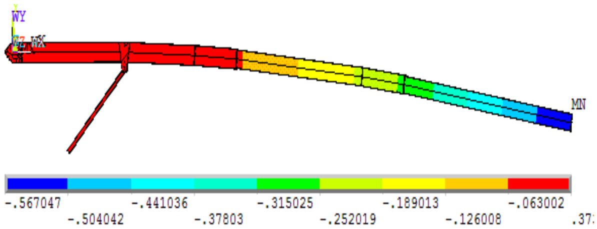


FIG 6 RESULT OF LARGE DEFORMATIN FEA

The results show that the maximum displacement of the jib in the luffing plane is 567.06 mm.

## *Reat Self Weight As Distributed Weight*

Both the difference of self-weight distribution and the impact of additional blending moment of each jib are carefully considered by treating self weight as distributed load.

### 1) *Theoretical Calculation of Distributed Weight*

By using the derived Eq. (9), the maximum displacement of jib in the luffing plane is 531.83 mm.

### 2) *Small Deformation FEA of Distributed Weight*

A finite element model is established as shown in Fig 4, and the constraints of degree-of-freedom are the same. In addition, the self weight of each jib is applied to each jib respectively, in distributed load method. The results of small deformation finite element analysis are shown in Fig 7.

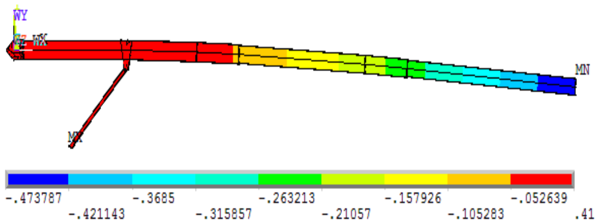


FIG 7 RESULT OF SMALL DEFORMATIN FEA

The result shows that the maximum displacement of the jib in the luffing plane is 473.88 mm.

### 3) *Large Deformation FEA of Distributed Weight*

Analysis method of this case is the same as described in large deformation FEA of concentrated weight, but the self weight is applied as distributed load along each jib length. Analysis calculation results are shown in Fig 8.

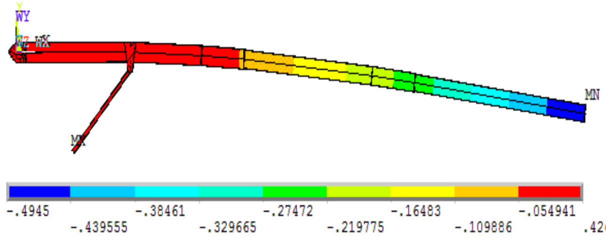


FIG 8 RESULT OF LARGE DEFORMATIN FEA

The results show that the maximum displacement of the jib in the luffing plane is 494.51 mm. The above analysis results are shown in Tab 2.

TABLE.2 RESULTS OF CALCULATION

| Models | 1) Concentrated | | | 2) Distributed | | |
|---|---|---|---|---|---|---|
| Method | Case 2.1.1 | Case 2.1.2 | Case 2.1.3 | Eq.(10) | Case 2.2.2 | Case 2.2.3 |
| Defle -ction /mm | 576.59 | 542.89 | 567.06 | 531.83 | 473.88 | 494.51 |
| Error% | -16.6 | -9.8 | -14.7 | -7.5 | 4.2 | 0 |

## Conclusions

Using the above six kinds of calculation methods, the displacements of telescopic jib in the luffing plane are gained respectively. Actually, for the working condition of fully extended jib, the bending deflection is very large and the bending moment caused by external axial force and self weight can not be ignored, so the calculation of deformation should be in accordance with the large deformation geometric nonlinear theory. The results of calculation of different methods are compared and evaluated, benchmarked against the result of large-deformation finite element analysis. From the results of Tab 2, it can be seen that:

1) By treating weight as concentrated weight, the two results calculated in equivalent-moment inertia method and in small deformation FEA of concentrated weight method are larger than the results by treating weight as distributed weight. This phenomenon that is consistent with the description in literature 3 and literature 8 is reasonable. Since the additional bending moment caused by jib nonlinear deformation is not considered by small deformation FEA, its result is smaller than others.

2) By treating weight as distributed weight, the result from the deduced formula and the result from the small deformation FEA of distributed weight are both close to the standard value. However, the small deformation FEA result is smaller than the standard value, which is risky for design, thus this method is not recommend in engineering.

3) Equivalent-moment inertia method is simple and practical, whose result is larger than standard value, leading to heavyweight telescopic jib. The result of Eq.(9) is more precise and reasonable. It is a science way to realize the lightweight and security of telescopic jib system.

### ACKNOWLEDGMENT

This paper is supported by the National Science-technology Support Projects for the 12th five-year plan (2011BAK03B0904).

### REFERENCES

Banerjee, A., Bhattacharya, B., Mallik, A. K., Large deflection of cantilever beams with geometric non-linearity: Analytical and numerical approaches [J]. Non-linear mechanics, 2008(43): 366~376.

Bu, Junyan. Nonlinear analysis research and application for jib system based on finite element method [D]. Dalian: Dalian University of Technology, 2008.

Cardoso, E. B., Benedito, Nuno M. B., et. Finite element analysis of thin-walled composite laminated beams with geometrically nonlinear behavior including warping deformation [J]. Thin-Walled Structures, 2009, 7: 1363~1372.

Guan, Yue, Li, Quanwang. From Castigliano's Second Theorem to discuss the relationship between calculation method of structural displacement. Mechanics in Engineering [J], 2012(Vol 34): 66~67.

Guo, Yimu, Tao, Weiming. Linear and nonlinear finite element method and application[M]. Beijing: CHINA MACHINE PRESS, 2003.

Hu Jinglong et al. Newton-coates method of structural displacement calculation. Mechanics in Engineering [J], 2008(Vol 30): 93~95.

Ji, Aimin, Zhang, Peiqiang, Peng, Duo, et al. Finite element analysis for local stability of telescopic jib of truck crane [J]. Agricultural Engineering Journal, 2004, 35(6): 48~51.

Li, Liankun. Structural Mechanics (4th ed) [M]. Beijing: Higher Education Press, 2009.

Liu, Yueyan. The application of Gauss quadrature method in the structural displacement calculation [J]. Architectural and Structural Design, 2010: 17~19.

Nayfeh, A.H., Emam, S. A., Exact solution and stability of postbuckling configurations of beams. Nonlinear Dynam, 2008(54): 395~408.

Raftoyiannis, Loannis G, Eromopoulos, John Ch., Stability of Tapered and Stepped steel Columns with Initial Imper-fections. Engineering Structures.2005, 27(8): 1248 ~1257.

Teng, Rumin, Liu, Kanyuan, Chen, Li. FEM calculation for hoisting performance of large-tonnage telescopic jib cranes [J]. CHINESE JOURNAL OF CONSTRUCTION MACAHINERY, 2011(6): 194~199.

Wang, Xin, Huang, Lin, Gao, Yuan, et al. Investigation into topological optimization for telescopic jib sections of cranes.[J]. CHINESE JOURNAL OF CONSTRUCTION MACAHINERY, 2007, 5(3): 288~292.

Xu, Kejin. Crane displacement calculation of variable cross-section components [J]. HOISTING AND CONVEYING MACHINERY, 1985(5): 19~25.

Xu, Gening. Mechanical equipment and metal structure Design. (2th ed)[M]. Beijing: China Machine Press, 2009.

Zhang, Hui, Cheng, Kai, Zhang, Cuifeng, et al. Calculation method of deflection for box jib [J]. Construction Machinery, 2012(9): 83~86.

[1]**Gening Xu**, (Hubei, 1955), male, Chinese, Dr. of Mechanical Engineering, Professor, field of research: Mechanical structures 3D-modeling, CAD/CAE system integration, Reliability failure analysis, Safety assessment,

Email: xugening@sina.com

[2]**Guangheng Gao**, (Shanxi, 1987), male, Chinese, A Graduate Student of Mechanical Design and Theory, field of research: Design method of the Great Length Jib Structure for Hydraulic Aerial Cage,

Email: ggh678@126.com